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ALL INDIA TEST SERIES CSE-2023

Candidate 's Information

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4. SUBJECT:- Paper 1 Full Length Test
5. DATE:- 27-08-23

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Q.NO	MARKS
1.	30
2.	
3.	32
4.	
5.	28
6.	
7.	27
8.	31½

TOTAL MARKS	149 / 250

EXAMINER SIGNATURE

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Q1(a) Planets revolve around the sun in elliptical orbits with sun at one focus.

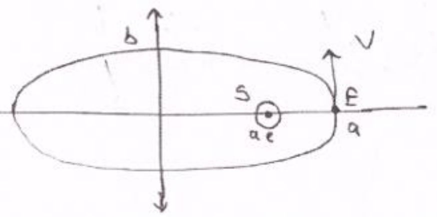

Fig 1. Planetary motion

Velocity at fall from 2a.

$$-\frac{GMm}{(2a)} = \frac{-GMm}{(a-ae)} + \frac{1}{2}mv^2.$$

Assuming earth is at E

Now $\frac{v^2}{2} = \frac{GM}{a}\left[\frac{1}{1-e} - \frac{1}{2}\right] \Rightarrow v^2 = \frac{GM}{a}\frac{(1+e)}{(1-e)}$

This is the velocity if object had fallen.

> As object is moving elliptically, it is bound i.e eccentricity < 1.

$\Rightarrow$ Total energy = $\frac{1}{2}$ PE $\Rightarrow E = \frac{-GMm}{(1-e)a}$ for pt. E.

$\Rightarrow KE = TE - PE \Rightarrow \frac{1}{2}mv^2 = +\frac{GMm}{2(1-e)a} - \frac{GMm}{(2a)}$

$\Rightarrow v^2 = \frac{GM}{a}\frac{(1+e)}{(1-e)}$ i.e. it is same as if it had fallen from height 2a.

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Q1 (b)

Planet revolves around sun due to gravitational force in an elliptical orbit

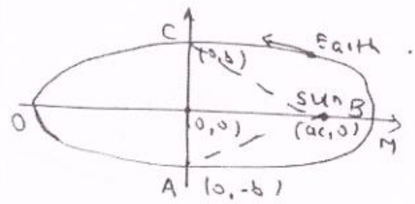

Fig 1. Planetary motion

We know, from Kepler's laws, Areal velocity $= V_A =$ constant $= J/2m$.

$\Rightarrow$ Time spent $= \dfrac{\text{Area swept}}{V_A}$,

$T_{(COA)} = \left(\dfrac{\pi ab}{2} + \dfrac{1}{2}(ae)(2b)\right)/V_A$ { Half-area of ellipse + Area of $\triangle ABC$

$T(AMC) = \left(\dfrac{\pi ab}{2} - \dfrac{1}{2}(ae)(2b)\right)/V_A$.

$\Rightarrow$ Total time $= \dfrac{\pi ab}{V_A} = T$. { On dividing by πab

$T_{(\text{minor axis})} = \left(\dfrac{1}{2} \pm \dfrac{e}{\pi}\right)T$

$\Rightarrow T(COA) = (1/2 + e/\pi)$; $T(AMC) = (1/2 - e/\pi)$

$\Rightarrow \Delta T = (2e/\pi)\,T = 0.0106$ (years)

$\Rightarrow$ $\boxed{\Delta T = 93.1 \text{ hrs}}$.

$\therefore$ Planet spends 93.1 hrs more between minor axis, away from the sun.

6½ / 10

Q1 (c)

Centre of Mass of a system is a point where whole mass of body is supposed to be concentrated.

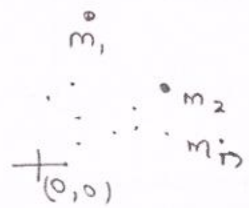

Fig 1. System of particles

> Consider system of n particles.

We know, velocity of COM $= V_{cm} = \dfrac{\sum m_i \vec{V_i}}{\sum m_i}$

We know, $\vec{v_i}' = \vec{v_i} - V_{cm}$.

where v_i' = velocity in COM frame.

$\Rightarrow \dfrac{dv_i'}{dt} = \dfrac{\vec{dv_i}}{dt} - \dfrac{dV_{cm}}{dt}$. } As COM is 0 momentum frame,

$\sum m_i v_i' = \sum m_i \dfrac{dv_i'}{dt} = 0$ always.

$\Rightarrow$ If $M \dfrac{dv_{cm}}{dt} \neq 0$, $\Rightarrow \sum m_i \dfrac{\vec{dv_i}}{dt} \neq 0$

$\Rightarrow$ $F_{external}$ exists.

Hence centre of mass is accelerated only if there is net external force on the system.

Q1 (d)

Michelson's Interferometer is pair of 2 identical mirrors placed perpendicular & can be used to estimate spectral width of source.

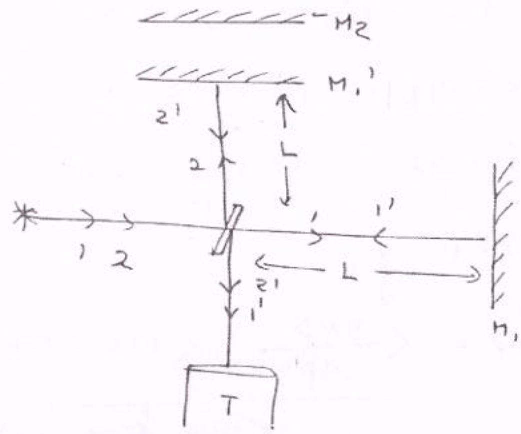

Fig 1. Michelson's interferometer

> If the source has spectral width, for pattern to disappear,

$2x = n\left(\lambda + \frac{\Delta\lambda}{2}\right) = \left(n+\frac{1}{2}\right)\lambda$

where x = mirror displacement

i.e. maxima of one λ coincides with minima.

On solving, $2x = \frac{\lambda^2}{\Delta\lambda} \Rightarrow$ $\Delta\lambda = 18 A^{0}$

Coherent length $= \frac{\lambda^2}{\Delta\lambda} =$ 0.2mm i.e. length till which phase relationship is maintained.

Government of India is establishing Michelson Interferometer at Hingoli to study Gravitational waves.

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Q1(e) Zone plate is specially designed diffracting plate which acts like convex lens with multiple focii

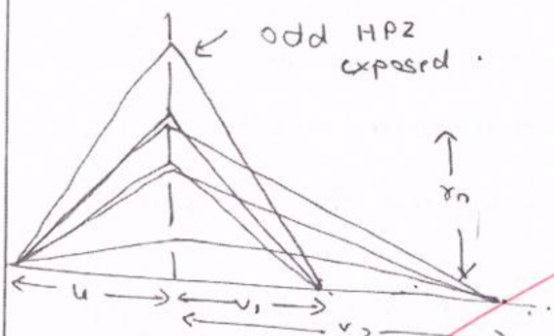

Fig 1. Zone plate

> It is observed that multiple images are formed. Using Fresnel's theory,

$$\langle y_n' \rangle = \left\langle \frac{n v \lambda}{(2p-1)} \right\rangle \langle 1+\cos\theta_n \rangle$$

⟨avg. distance⟩

where $\langle y_n' \rangle$ = contribution of each Half-period zone (HPZ).

As $\langle y_n' \rangle = \frac{\langle y_n \rangle}{(2p-1)}$, each HPZ is divided into $(2p-1)$ HPZ.

$\Rightarrow$ $y_1 = S_1 - S_2 + S_3$; $y_3 = S_6 - S_7 + S_8$. } For p = 2.

$\therefore$ $y = \sum y_n = \frac{\langle y_n \rangle}{(2p-1)}$. Hence, we get multiple focii as $f = \frac{r_n^2}{n\lambda(2p-1)}$.

For $r_n = 1\,\mu m$, $\lambda = 6000\,\mathring{A}$, $n = p = 1$,

$f = 1.667\,m$

Zonal plate can be used as convex lens for any type of EM wave.

6½/10

Q3 (a)

Einstein's postulate of special theory of relativity that velocity of light is absolute constant gave birth to concept of space time

$$\Delta S = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2 - c^2(t_1-t_2)^2}$$

$$\Rightarrow \Delta S = \sqrt{(5c)^2 - c^2(3)^2} = 4c$$

$\therefore$ Space time interval $= \boxed{4c}$.

(i)

Fig 1. Two inertial frames

Let $A = (x, y, z, t)$
$= (0, 0, 0, 0)$
$B = (x, y, z, t)$
$= (5c, 0, 0, 3)$

Using Lorentz transform,

$$t_1' = \alpha\left(t_1 - \frac{x_1 v}{c^2}\right) \quad ; \quad t_2' = \alpha\left(t_2 - \frac{x_2 v}{c^2}\right) \Bigg\} \; t_i = \text{time in } S'$$

If $t_1' = t_2'$,

$$t_1 - \frac{x_1 v}{c^2} = t_2 - \frac{x_2 v}{c^2} \Rightarrow (x_2 - x_1)\frac{v}{c^2} = t_2 - t_1$$

$$\Rightarrow (5c)\frac{v}{c^2} = 3 \Rightarrow v = \frac{3}{5}c = \boxed{1.8 \times 10^8 \text{ m/sec}}$$

(ii) $t_1' = \alpha\left(t_1 - \frac{x_1 v}{c^2}\right)$; $t_2' = \alpha\left(t_2 - \frac{x_2 v}{c^2}\right)$

Given $t_2' - t_1' = 1$

$\Rightarrow \alpha\left[t_2 - t_1 - \frac{v}{c^2}(x_2 - x_1)\right] = 1$

$\Rightarrow \alpha\left[3 - \frac{v}{c^2}(5c)\right] = 1$

$\Rightarrow 3 - \frac{5v}{c} = \sqrt{1 - \frac{v^2}{c^2}}$ { Let $v/c = x$.

$\Rightarrow 3 - 5x = \sqrt{1-x^2} \Rightarrow 9 - 30x + 25x^2 = 1 - x^2$

$\Rightarrow 26x^2 - 30x + 8 = 0$.

$\Rightarrow x = 0.73$ or $x = 0.418$.

$\therefore$ For $v/c = 0.73$, $\Delta t = -ve \Rightarrow$

$\boxed{v = 0.42c}$

13/20

(iii) Given $t_1' - t_2' = 1$

$\Rightarrow \alpha\left[(t_1 - t_2) - \frac{v}{c^2}(x_1 - x_2)\right] = 1$

$\Rightarrow \alpha\left[(-3) - \frac{v}{c^2}(-6c)\right] = 1$

$\Rightarrow -3 + 5v/c = \sqrt{1 - v^2/c^2}$ Let $v/c = x$.

$\Rightarrow (5x - 3) = \sqrt{1-x^2}$

$\Rightarrow 25x^2 - 30x + 9 = 1 - x^2$.

$\Rightarrow 26x^2 - 30x + 8 = 0 \Rightarrow x = \underline{0.42c}$.

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Answer Questions in NOT MORE THAN the Word Limit specified for each in the Parenthesis.
(Specimen Answer Booklet - For Practice Purpose Only)

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or $x = 0.73c$. But for $x = 0.42c$, Δt is -ve

$\Rightarrow \boxed{v = 0.73c}$.

(iv). Maximum time interval $= \Delta t'$

$= \alpha \left[t_2 - t_1 - \frac{v}{c^2}(x_2 - x_1) \right]$

$= \alpha \left[3 - 5v/c \right] = dt$ } Let $v/c = x$.

$\Rightarrow dt = \frac{3-5x}{\sqrt{1-x^2}}$. } For maxima, $\frac{dt}{dx} = 0$.

$\Rightarrow 5\sqrt{1-x^2} = \frac{x(3-5x)}{\sqrt{1-x^2}} \Rightarrow 5 - 5x^2 = 3x - 5x^2$.

$\Rightarrow x = 5/3$. But this is not possible as v has to be $< c$.

$\therefore$ The limit would be when $v \to c$ as at that point $\Delta t' \to \infty$

However, this is practically unrealizable.

Q.3 (b)

Damped harmonic oscillator has a damping force in addition to restoration force.

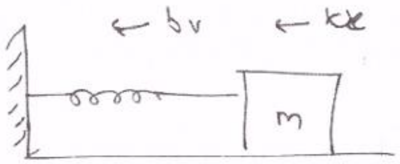

Fig 1. Damped Oscillator

Eq. of motion is given by:-

$$m\ddot{x} + b\dot{x} + kx = 0.$$

$$\Rightarrow \ddot{x} + 2c\dot{x} + \omega_0^2 x = 0.$$

Where $2c = b/m$ = damping const.

$\omega_0 = \sqrt{\frac{k}{m}}$ = natural frequency.

Let $x = Ae^{\alpha t}$. $\Rightarrow (\alpha^2 + 2c\alpha + \omega_0^2) = 0$.

Here 3 cases arise based on damping:-

Case-I :- Overdamped i.e. $c > \omega_0$.

$$\Rightarrow x = Ae^{-ct}\left[c_1 e^{\sqrt{c^2-\omega_0^2}\,t} + c_2 e^{-\sqrt{c^2-\omega_0^2}\,t}\right]$$

The oscillations die exponentially.

Case-II :- Critical i.e. $c = \omega_0$,

$$x = (c_1 + c_2 t)(Ae^{-ct}).$$

Case-III :- Underdamped i.e. $c < \omega_0$.

$$x = Ae^{-ct}\sin\left(\sqrt{\omega_0^2 - c^2}\,t + \phi\right)$$

i.e. oscillations reduce sinusoidal - exponential way.

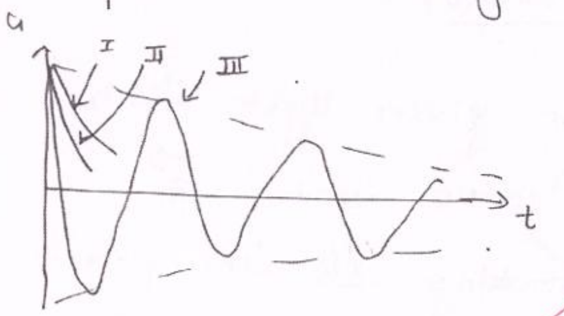

Fig 2. 3 cases Amplitude (a) v/s t

> Hence, damping affects the amplitude & frequency - More the damping, faster the amplitude fall.

Pendulum. In case of pendulum, in over-damped case, oscillations would fall rapidly & pendulum would come to halt.

> In case of critical case, assuming no other force, pendulum would keep swinging as centrifugal force = damping.

> Damped oscillations is used in dead beat galvanometer to stabilize needle.

9½/15

Q.3 (c)

Polarization is shape & locus of tip of electric field vector.

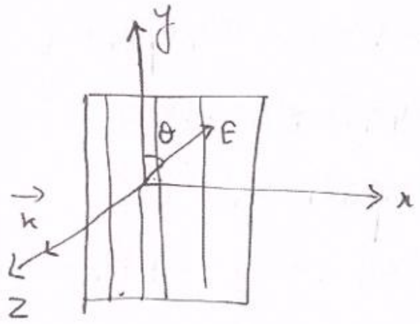

Fig 1. Linear polarized light

For given light with electric vector $\vec{E}$ making 2θ with y-axis,

$$\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} - \frac{2E_xE_y}{ab}\cos\phi = \sin^2\phi.$$

Now if $\phi = (2n-1)\pi/2$ or $\Delta = \frac{\lambda}{2\pi} = (2n-1)\frac{\lambda}{4}$, we get circular light.

Producing in laboratory :- Pass unpolarized light through Nicole prism ⟶ we get plane polarized light (e-ray) ⟶ pass through Quarter Wave Plate (QWP) ⟶ Circular polarized light.

★ CP light passed through QWP

> CP light has path difference of

$(2n-1)\lambda/4$. when passed through QWP, additional path difference of $\lambda/4$ will be introduced $\Rightarrow \Delta = (n\lambda/2) \Rightarrow \phi = n\pi$

i.e. light will become linearly polarized.

RCP - LCP Distinction

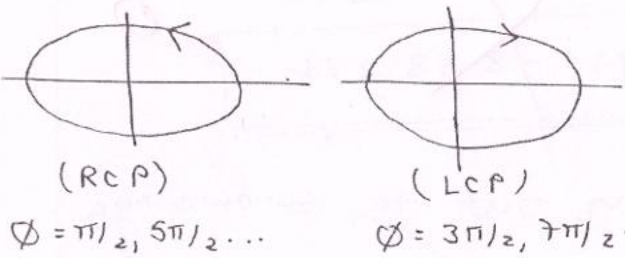

Fig 2. RCP & LCP

(1) Pass through optically active medium :- Like sucrose solution. As it would react differently with RCP & LCP due to helicity, it can be distinguished.

(2) Polarizer - analyzer :- Pass through QWP, we get plane polarized light, path difference can be used to measure inclination with optical axis (θ)

Usually, Nicole's Prism is used as both analyzer & polarizer in such experiments

Q9 (a)

Entropy (S) is the measure of disorderliness of system & is given by:-

$\Delta S = \int \frac{dQ}{T}$. Here $dQ = -I^2 R dt$

$\Rightarrow \Delta Q = -I^2 R \Delta t$

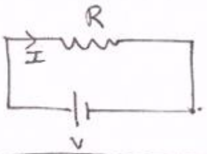

Fig 1. Resistor heating

As it is heated at constant temperature

No state change

$\Rightarrow \Delta S_{sys} = -\frac{(10)^2 (25)(1)}{300}$

$\Rightarrow \boxed{\Delta S_{sys} = -8.33 \ J/k}$ = 0

(ii) This heat is given out to surroundings

$\Rightarrow \Delta S_{surr} = +8.33 \ J/k$

$\Rightarrow \Delta S_{univ} = \Delta S_{sys} + \Delta S_{surr} = \boxed{0}$.

This is consistant with law of entropy that entropy of universe is always ≥ 0

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Q5 (b)

Polarization is dipole moment per unit volume

Given :- ~~Polarization~~ Dipole moment $(\vec{p}) = k\vec{r}$.

We know, for a polarized body,

σ_b = Surface charge density = $\vec{P}\cdot\hat{n}$

$\Rightarrow$ ~~σ/r~~ where $\vec{P} = (4/3\pi a^3)^{-1} \times \vec{p}$.

$\Rightarrow \left.\frac{k\cdot r\hat{r}}{4/3\pi a^3}\cdot(\hat{r})\right]_{r=a} \Rightarrow \boxed{\sigma_b = \frac{3k}{4\pi}\left(\frac{1}{a^2}\right)}$

ρ_b = Bound volume charge density = $-\vec{\nabla}\cdot\vec{P}$.

$\Rightarrow -\vec{\nabla}\cdot\left(\frac{k}{4/3\pi a^3}r\hat{r}\right) \Rightarrow k' \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 r)$

$\Rightarrow$ ~~k'~~ $\rho_b = -\frac{3k}{4\pi}\left(\frac{1}{a^3}\right)(2) = \boxed{-\frac{3k}{2\pi a^3}}$.

(ii) Total charge $= \int \sigma_b \cdot da + \int \rho_b \cdot d\tau$

$= \boxed{0}$.

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Polarization leads to differential displacement of charged particles because of which we get σ_b & ρ_b

(Q5) (c)

Poynting vector is also known as energy intensity i.e. energy per unit per unit time.

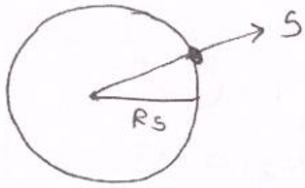

Fig 1. Poynting vector(s)

$\Rightarrow S = \frac{Energy}{Area - time} = \frac{Power}{Area.}$

Given :- $R_S = 7 \times 10^8 m$

$\Rightarrow$ Area $= 6 \cdot 16 \times 10^{18} m^2$

$\Rightarrow$ $S = 6 \cdot 17 \times 10^7 W/m^2$

Poynting vector from sun is used to estimate solar insolation on earth.

Q5 (d)

Heat capacity is the amount of heat required to raise temperature of unit mass of substance by unit temperature.

C_V = Heat capacity at constant volume $= \left(\frac{\partial U}{\partial T}\right)_V$.

To show, $\left(\frac{\partial C_V}{\partial V}\right)_T = 0 \Rightarrow \left[\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial T}\right)_V\right]_T = 0$.

As dU is a perfect differential because, U is a state function.

$$\left[\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial T}\right)_V\right]_T = \left[\frac{\partial}{\partial T}\left(\frac{\partial U}{\partial V}\right)_T\right]_V$$

We know, $dU = TdS - PdV$.

$$\Rightarrow \left[\frac{\partial}{\partial T}\left[\frac{\partial}{\partial V}(TdS - PdV)\right]\right] = \frac{\partial}{\partial T}\left[T\left(\frac{\partial S}{\partial V}\right)_T - P\right]$$

$= T\left(\frac{\partial^2 P}{\partial T^2}\right)_V$

From Maxwell's eq, $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

Also, for ideal gas, $PV = RT$, $\Rightarrow \left(\frac{\partial P}{\partial T}\right) = \frac{R}{V}$

$$\Rightarrow \left(\frac{\partial C_V}{\partial V}\right)_T = \frac{\partial}{\partial T}\left(\frac{RT}{V} - P\right) = 0$$

This is because in ideal gas internal energy is independent of volume.

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Q5 (c)

$\Delta H, \Delta F, \Delta G$ are thermodynamic potentials which provide various parameters like work done, spontaneity, etc.

Define Adsorption & then say $\Delta H, \Delta S$ decreases

We know,

$\Delta H = \Delta(U+PV) = TdS + VdP$ — (1)

$\Delta F = \Delta(U-TS) = -PdV - SdT$ — (2)

$\Delta G = \Delta(H-TS) = VdP - SdT$ — (3)

As it is isothermal process, $dT = 0$

As it is spinatenous, $\Delta G < 0 \Rightarrow VdP < 0$

As the gas is adsorped, $\Delta S < 0 \Rightarrow \Delta H < 0$.

Assuming ideal gas, $PV = RT \Rightarrow PdV + VdP = RdT = 0$.

As $VdP < 0$, $PdV > 0 \implies \boxed{\Delta F < 0}$

$\therefore$ All three – $\Delta G, \Delta H, \Delta F$ are negative & this is the reason for spontaneity.

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Q7 (a)

Resistor - capacitor circuit shows exponential growth of charge, current, etc.

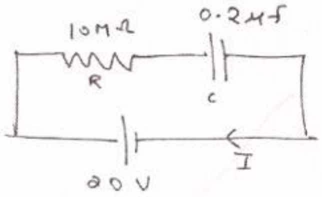

Fig 1. Circuit

We know, from Kirchoff's law,

$V = IR + \frac{Q}{C}$. As $I = \frac{dQ}{dt}$,

$\frac{dQ}{dt} + \frac{Q}{RC} = \frac{V}{R}$. } On solving,

$Q = CV\left(1 - e^{(-t/RC)}\right)$

(i) Rate of growth of charge $= \frac{dQ}{dt} = \frac{V}{R} e^{-t/RC} = I$

$\Rightarrow \frac{dQ}{dt} = 2 \times 10^{-6} e^{-t/2}$ A

(ii) Energy stored in capacitor.

We know, $E = \frac{Q^2}{2C} \Rightarrow \frac{dE}{dt} = \frac{Q}{C}\frac{dQ}{dt}$.

$\Rightarrow \frac{dE}{dt} = V\left(1 - e^{-t/RC}\right)\frac{V}{R}e^{-t/RC}$

$\Rightarrow E = \frac{V^2}{R}\int_0^t e^{-t/RC} - e^{-2t/RC} \, dt$.

$\Rightarrow E = \frac{V^2}{R}\left[e^{-t/RC}(-RC) - e^{-2t/RC}\left(\frac{-RC}{2}\right)\right]_0^t$

$\Rightarrow E = CV^2 \left[\frac{e^{-2t/RC}}{2} - e^{-t/RC} \right]_0^t$

$\Rightarrow \boxed{E = CV^2 \left[\frac{e^{-2t/RC}}{2} - e^{-t/RC} + \frac{1}{2} \right]}$

For $t = 2$, $E_{cap} = CV^2 \left[\frac{e^{-2}}{2} - e^{-1} + \frac{1}{2} \right]$

$\Rightarrow E_{cap} = 0.2\, CV^2 = \boxed{15.98\ \mu J}$

(iii) Heat in resistor $= \int_0^t I^2 R\, dt$

$\Rightarrow E_{resistor} = \int_0^t \frac{V^2}{R^2} e^{-2t/RC}\, dt\, (R)$

$\Rightarrow E_r = \frac{V^2}{R} \left[e^{-2t/RC} \cdot \left(\frac{-RC}{2}\right) \right]_0^t$

$\Rightarrow \boxed{E_r = \frac{-1}{2} CV^2 \left[e^{-2t/RC} - 1 \right]}$

At $t = 2$ sec, $E_r = 138.3 \times 10^{-6}$ J

(iv). Energy delivered by battery

$\Rightarrow (E_r + E_{cap})_{2 secs}$

$\Rightarrow$ Energy delievered in 2 secs

$= \boxed{154.23\ \mu J}$

Such circuits are usually found in DC motors.

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Q 7 (b)

Electric & magnetic field are two types of fields associated with a propogating electromagnetic wave. Maxwell & Faraday tried to explain them as follows :-

Faraday's Concepts. Explains the observed induced current due to changing magnetic flux as. $e = -\frac{d\phi}{dt}$ } e = potential, ϕ = flux.

> He considered these 2 as separate fields with one leading to another.

Maxwell's Concepts Integrated both electric & magnetic field & showed that

(i) E, B, k form right handed triad.

(ii) Gave Maxwell's differential equation: based on which electric & magnetic field of any material can be found.

(iii) Under ~~Coulo~~ Lorentz gauge, electric &

magnetic field combine to give single equation $\Box^2 A_\mu = -\mu_0 J_\mu$.

> Hence, while Faraday treated fields separately, Maxwell treated them together. Faraday only found relation between 2 fields, Maxwell gave a way to find their values.

Displacement Current Imaginary current which accounts for changing electric field making total current solenoidal.

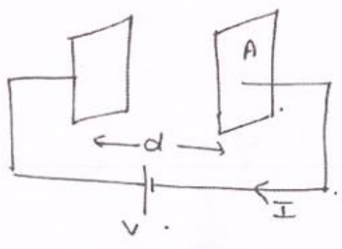


Fig 1. Parallel Capacitor

We know, $I_D = A \frac{\partial \vec{D}}{\partial t}$.

$\Rightarrow I_D = A \varepsilon \frac{\partial \vec{E}}{\partial t}$.

In parallel plate capacitor,

$E = \frac{V}{d} \Rightarrow I_D = \frac{A\varepsilon}{d} \frac{\partial V}{\partial t}$

Now $Q = CV \Rightarrow I_D = \frac{\varepsilon A}{d} \left(\frac{1}{C}\right) \frac{dQ}{dt}$.

We know for parallel plate capacitor,

$C = \frac{\varepsilon_0 A}{d} \Rightarrow I_D = \frac{dQ}{dt} = I$.

i.e. in parallel plate capacitor, displacement current is actually charging current I.

Usually, displacement current is very small & is not observed.

Q.7 (c)

Dispersion is broadening of spectrum due to variation of refractive index (n) with wavelength (λ).

Normal Dispersion

> Condition :- $dv_p/d\lambda > 0 \Rightarrow \frac{dn}{d\lambda} < 0 \Rightarrow \frac{dn}{d\omega} > 0$

> In normal dispersion, $\omega_j >>> \omega \Rightarrow j\gamma\omega \approx 0$.

$$\Rightarrow n = 1 + \frac{Ne^2}{2m\varepsilon_0} \sum \frac{n_j}{\omega_j^2 - \omega^2}.$$

We know, $\omega = \frac{2\pi c}{\lambda}$.

Substituting & expanding binomially, we get;

$$n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots$$

(Cauchy formula) $\Rightarrow dn/d\lambda < 0$.

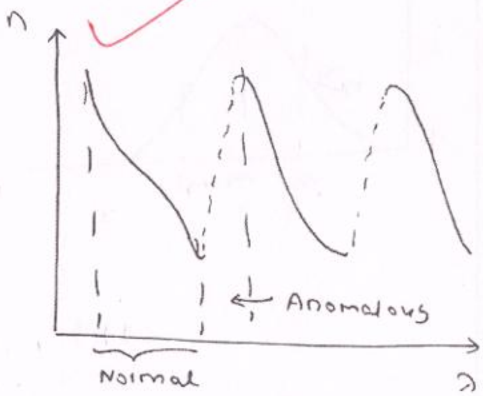


Fig 1. n v/s λ

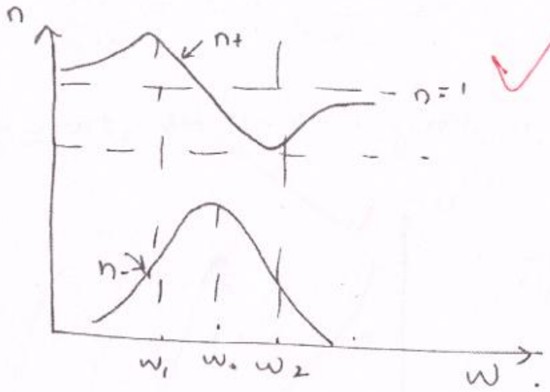
Anomalous Dispersion

> Condition $dv_p/d\lambda < 0$ i.e. red light travels slower.

Let $n = n_+ + j n_-$

$$\Rightarrow n_+ = 1 + \frac{Ne^2}{2m\varepsilon_0} \sum \frac{n_j (\omega_j^2 - \omega^2)}{[(\omega_j^2 - \omega^2)^2 + \gamma^2 \omega^2]}$$

$$n_- = \frac{Ne^2}{2m\varepsilon_0} \sum \frac{n_j (\gamma^2 \omega^2]}{[(\omega_j^2 - \omega^2)^2 + \gamma^2 \omega^2]}.$$



Here 2 graphs can be drawn for n_+ & n_- wrt to ω

It is because of the dispersion that a white light is spread across colors by medium like thin film.

(Please do not write anything except the question number in this space) कृपया इस स्थान में प्रश्न संख्या के अतिरिक्त कुछ न लिखें।

UPSC

Answer Questions in NOT MORE THAN the Word Limit specified for each in the Parenthesis.
(Specimen Answer Booklet - For Practice Purpose Only)

उम्मीदवारों को इस हाशिए में नहीं लिखना चाहिए
Candidates must not write on this margin

Q 8 (a)

Refrigerator can be assumed as Carnot cycle operating in reverse cycle.

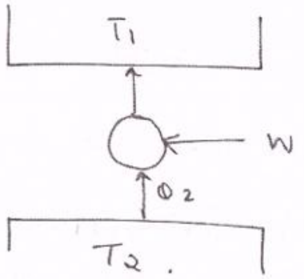


Fig 1. Refrigeration cycle

We know, most efficient cycle is reversible carnot such that COP

$$= \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}.$$

Assuming ice is at 0°C,

$$\text{Energy lost} = Q_2 = L\left(\frac{dM}{dt}\right)$$

$$\Rightarrow Q_2 = 80 \times 4.18 \times \frac{36 \times 10^3}{3600} = \boxed{3344 \text{ J/sec}}$$

Here $T_2 = 273k$, $T_1 = 303k$.

$$\Rightarrow COP = \frac{3344}{W} = \frac{273}{30}.$$

$$\Rightarrow \boxed{W = 367.5 \text{ J/sec}}$$

∴ Minimum power required to prevent ice from melting at 0°C is $\boxed{367.5 \text{ W}}$.

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Q 8 (b)

Clausius Clapeyron equation gives the variation of boiling /melting point with pressure

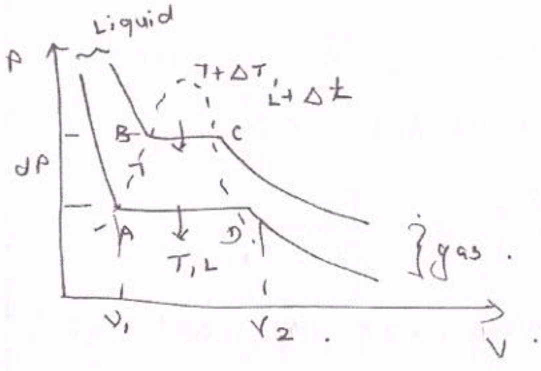

Fig 1. Isotherms at two temperatures

Consider 2 isotherms as shown. We construct imaginary adiabats AB & CD to form a closed cycle ABCD.

We know, for reversible cycle, $\eta = 1 - \frac{T}{T+\Delta T} = 1 - \frac{L}{L+\Delta L}$

$\Rightarrow LT + T\Delta L = LT + \Delta T(L) \Rightarrow \Delta L = \frac{L\Delta T}{T}$

We know $\Delta L = \oint dw = dP(V_2 - V_1)$

$\Rightarrow dP(V_2 - V_1) = \frac{L\Delta T}{T}$

$$\Rightarrow \boxed{\left(\frac{\partial P}{\partial T}\right) = \frac{L}{T(V_2 - V_1)}}$$ Clausius Clapeyron eq.

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Specific heat of substance usually remains constant with temperature but when it varies, we vary both P & V to keep vapour saturated. Using entropy, we get,

$$\frac{dL}{dT} - \frac{L}{T} = (C_2)_{sat} - (C_1)_{sat}$$ { C_2 = saturated vapour, C_1 = saturated liquid.

Here, $\frac{dL}{dT} = -0.715$ — (1).

$$\frac{L}{T}\Big]_{T=273K} = \frac{800 - 0.715(273)}{273} = 2.215$$

$\Rightarrow \frac{dL}{dT} - \frac{L}{T} = -2.93$. We know, $(C_1)_{sat} = 1\ cal/gm$.

$\Rightarrow (C_2)_{sat} =$ **$-3.93\ cal/gm$** i.e. specific heat is negative. This specific heat is composed of 2 elements :-

Q_1: Heat needed to raise temperature.

Q_2: Heat taken out to keep vapor saturated.

If $(C_2)_{sat} < 0$, more heat is taken out $(Q_2 > Q_1)$ than ~~to~~ given to raise temperature

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Q8 (c)

Bosons are full integral spin indistinguishable particles.

> To find no. of bosons in energy ε, we need to find no. of microstates (Ω) available such that :-

(i) Equal apriori probability.

(ii) No restriction of no. of particles /cell

(iii) Particles are indistinguishable.

$\Rightarrow (\Omega) = \prod_i {}^{g_i+n_i-1}C_{n_i}$ } g = # cells, n_i = # particles.

$\Rightarrow \ln\Omega = \sum \ln(g_i+n_i-1)! - \ln(n_i)! - \ln(g_i-1)!$

We know $S = \ln\Omega \Rightarrow dS = d(\ln\Omega) = 0$ at equilibrium.

> Also, $d\sum n_i = 0$, & $d\sum n_i\varepsilon_i = 0$ } # particles & energy is constant.

> Using sterling approximation $\ln n! = n\ln n - n$,

& Lagrange multiplier, we get,

$$d(\ln \Omega) = \sum \left(\ln\left(\frac{g_i + n_i}{n_i}\right) + \alpha - \beta \epsilon_i \right) dn_i = 0$$

$$\Rightarrow \frac{n}{g} = f(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/kT} - 1}$$

$$\Rightarrow \boxed{n = \frac{g}{e^{(\epsilon-\mu)/kT} - 1}}$$

Here μ = chemical potential of bosons.

> To find μ, no. of bosons can be calculated theoretically. As $\epsilon \to \mu$, $n \to \infty$

$\Rightarrow$ We can increase energy of Bosons via increasing temperature to check when it achieves large value.

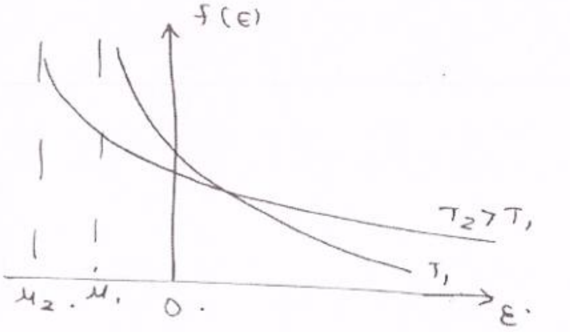

Fig 1. Bose-Einstein distribution

> At small temp, $A \simeq 0$.

Recently, Bose-Einstein experiment was conducted on ISS to observe it in zero-gravity.

7½ + 3 / 20 = 12½ / 20